Fundamental Interactions. Part 2

Plan of lectures

1. Strong interactions
2. Neutrino oscillationa
3. Phase Transitions
4. Gravitation
5. Extra dimensions

Previously

Choose $\mathcal{L}[\phi, \partial_\mu \phi]$ Bring eq. Lagrange - Euler

Fields: scalar (real and complex), vector, spinor

Potentials for different types of fields

Get an example

Fields ϕ transform under fundamental representation of G group.
(ϕ - common symbol for fermions and Higgs field)

Set of matrixes U

$$U \in G$$

$$\varphi' = U\varphi;$$

What is fundamental representation?

$$V_\mu' = U^{-1} V_\mu U + U^{-1} \partial_\mu U$$

Symmetrizaton of theory

Consider the lagrangian density $\mathcal{L}[\phi, \partial_\mu \phi]$ h is invariant under the following tranformations

$$\phi' = U(\theta^A)\phi \qquad (A = 1, 2, ..., D)$$

Why do we need symmetries?

$$L(\phi) - L(\phi') = 0$$

This condition leads to the conservation of some quantities Q:

$$\frac{dQ}{dt} = 0$$

Properties of U matrixes. Invariance of typical term

$$\phi_a \phi_a = U_{ab}\phi'_b U_{ac}\phi'_c = U^T_{ba} U_{ac}\phi'_b \phi'_c \overset{?}{=} \phi_b \phi_b$$

$$U^T U = 1$$

To satisfy the last equality matrix must be unitary

For small parameters $U(\theta^A) = 1 + ig\sum_A \theta^A T^A$

T^A - Generators of the group G in some representation

$[T^A, T^B] = iC_{ABC}T^C$ Structure constants defined by group G

What are the constants for
U(1) and SU(2) groups?

Gauge theories

In case of local symmetry, $\theta^A = \theta^A(x_\mu)$.

One should introduce a "long" derivative and matrix-field V

(*) $D_\mu = \partial_\mu + ig\mathbf{V}_\mu$

Constant

Under gauge transformation

$$\mathbf{V}'_\mu = U\mathbf{V}_\mu U^{-1} - (1/ig)(\partial_\mu U)U^{-1}$$

For convinience let's define $V_\mu = \sum_A T^A V_\mu^A$ since V is the matrix, acting on ϕ

V_μ^A are a set of D gauge fields

Number of gauge fields – interaction carriers EQUALS to number of symmetry group parameters.

How does V_μ^A transform under small shifts?

With θ^A = const field V transforms as adjoint representation of the group:

For small shifts

$$\mathbf{V}'_\mu = U\mathbf{V}_\mu U^{-1} - U\partial_\mu U^{-1} \longrightarrow V'^C_\mu = V^C_\mu - gC_{ABC}\theta^A V^B_\mu \quad -\partial_\mu \theta^B T^B$$

Proof $V_\mu = \sum_A T^A V_\mu^A$ and $U(\theta^A) = 1 + ig\sum_A \theta^A T^A$

$$V'_\mu = (1 + ig\theta^A T^A)V_\mu(1 - ig\theta^B T^B) = V_\mu + ig\theta^A T^A V_\mu - ig\theta^A T^A V_\mu =$$

$$= V_\mu + ig\theta^A\left[T^A V_\mu\right] = V_\mu + ig\theta^A\left[T^A T^B\right]V_\mu^B = V_\mu + ig\theta^A iC_{ABC}T^C V_\mu^B$$

Problem to solve

+ take into account $U\partial_\mu U^{-1}$

$$U\partial_\mu U^{-1} = (1 + ig\theta^A T^A)\partial_\mu(1 - ig\theta^B T^B) = -ig\partial_\mu \theta^B T^B$$

It is easy to get

$$(D_\mu\phi)' = U(D_\mu\phi)$$

and hence the original Lagrangian is invariant under local transformations

For fields to acquire a dynamic sense, it is necessary to introduce Lagrangian of the free gauge field

$$\mathcal{L}_{\mathrm{YM}} = -\frac{1}{4}\sum_A F^A_{\mu\nu}F^{A\mu\nu}$$

$$F^A_{\mu\nu} = \partial_\mu V^A_\nu - \partial_\nu V^A_\mu - gC_{ABC}V^B_\mu V^C_\nu$$

$$\mathbf{F}'_{\mu\nu} = U\mathbf{F}_{\mu\nu}U^{-1}$$

The simplest case

$$U[\theta(x)] \;=\; \exp[ieQ\theta(x)] \longrightarrow V'_\mu = V_\mu - \partial_\mu\theta(x)$$

$$C_{ABC} = 0$$

One can see, that V – is a vector potential of electromagnetic field

To sum up:

1. Set the group of symmetry
2. Introduce new fields which transform under fundamental representations
3. Introduce gauge fields
4. Choose a Lagrangian
5. Check for experimental issues

Strong interactions

Charged nucleus contains many protons at close range.
To neutralize the repulsion there could be introduced a new short-range force

Electro-magnetic interactions. – group U(1), boson - photon
electro-magn. interactions + weak – group U(1)xSU(2), bosons – photon + W,Z – bosons
electro-magn. interactions + weak + strong – group U(1)xSU(2)xSU(3), bosons – photon + W,Z–bosons + gluons

$$\psi \xrightarrow{SU(2)} \psi_i (i=1,2) \xrightarrow{SU(3)} \psi_{i,a} (i=1,2;\ a=1,2,3)$$

leptons $\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L ; e_R$ Mass term ~ $\bar{l}_L l_R$

That is why neutrino is massless

SU(3) – singlets

quarks $Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L ; d_R ; u_R \longleftarrow$ mass #0 + strong interactions

SU(3) – triplets

Name	Symbol	Mass (MeV/c^2)*	J	B	Q	I_3
First generati						
Up	u	1.7 to 3.3	$\frac{1}{2}$	$+\frac{1}{3}$	$+\frac{2}{3}$	$+\frac{1}{2}$
Down	d	4.1 to 5.8	$\frac{1}{2}$	$+\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{2}$
Second genera						
Charm	c	$1{,}270\ ^{+70}_{-90}$	$\frac{1}{2}$	$+\frac{1}{3}$	$+\frac{2}{3}$	0
Strange	s	$101\ ^{+29}_{-21}$	$\frac{1}{2}$	$+\frac{1}{3}$	$-\frac{1}{3}$	0
Third generat						
Top	t	172,000 ± 900 ± 1,300	$\frac{1}{2}$	$+\frac{1}{3}$	$+\frac{2}{3}$	0
Bottom	b	$4{,}190\ ^{+180}_{-60}$	$\frac{1}{2}$	$+\frac{1}{3}$	$-\frac{1}{3}$	0

J = total angular momentum, B = baryon number, Q = electric charge, I_3 = isos

Strong interactions

Group of symmetry– SU(3)
Particles – quarks (fermions) – transformed under fundamental representation of the SU(3) group
Gauge fields – gluons - transformed under adjoint representation of the SU(3) group. Higgs field - SU(3) invariant

Lagrangian of strong interactions MUST have a form of

$$\mathcal{L} = -\frac{1}{4}\sum_{A=1}^{8} F^{A\mu\nu}F^{A}_{\mu\nu} + \sum_{j=1}^{n_f} \bar{q}_j (i\not{D} - m_j) q_j$$

$$\not{D} = D_\mu \gamma^\mu$$

Matter field– vector with 3 components
Gauge field - $n^2 - 1 = 8$ components (number of parameters)

$$D_\mu = \partial_\mu - ig_s V_\mu$$

$$V_\mu = \sum_{A=1}^{8} V_\mu^A t^A; \quad t^A = \frac{\lambda^A}{2}$$

Fermions of Standard Model

Lepton sector

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, e_R \quad \left| \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \mu_R \quad \right| \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L, \tau_R$$

Quark sector

3 generations

$$\begin{pmatrix} u \\ d \end{pmatrix}_L u_R, d_R \quad \left| \begin{pmatrix} c \\ s \end{pmatrix}_L c_R, s_R \quad \right| \begin{pmatrix} t \\ b \end{pmatrix}_L t_R, b_R$$

3 colors– fund. Representation of SU(3)

Baryons – 3 quarks , uud (proton) и udd (neutron).
Мезоны – 2 quarks (main) , $u\bar{d}$+ meson), $\frac{u\bar{u} - d\bar{d}}{\sqrt{2}}$ (π_0 meson)

Unification of notations

Fundamental representation SU(3) (3 colors)

$$\overbrace{U_n = u_R, c_R, t_R \qquad Q_n = \begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L}$$

$$D_n = d_R, s_R, b_R$$

$\longleftarrow q = +2/3$

$\longleftarrow q = -1/3$

Right,
Scalars SU(2) – singlets

Left,
fund.representation SU(2) – duplets

Number of colors *n=3*

Interaction Lagrangian of quarks and gauge fields

$$L \,\square\, \bar{\psi}_A \gamma_\mu \partial_\mu \psi_A$$

Introduction of “long” derivative mixes the components of the fields which differ by group index

Try to constract Lagrangian by yourself

$$\Delta L_s = i\bar{Q}_{Ln}\hat{D}Q_{Ln} + i\bar{U}_{Rn}\hat{D}U_{Rn} + i\bar{D}_{Rn}\hat{D}D_{Rn} - \frac{1}{2}TrV_{\mu\nu}V^{\mu\nu}$$

$$D_\mu = \partial_\mu - ig'\frac{Y}{2}B_\mu - igT_W^i A_\mu^i - ig_s T_s^a V_\mu^a$$

$$T_s^a = \lambda^a / 2$$

-Generators of the group SU(3) → mixing the colors

$$T_w^i = \sigma^i / 2$$

-- Generators of the group SU(2) → mixing up and down quarks

Lagrangian stays diagonal in generations

$$uA_\mu^i d, \quad uA_\mu^i u$$

-- There appears terms not diagonal in color but diagonal in generations

Lagrangian of quark interaction (one generation) with Higgs (without SU(3) because of Higgs)

Keep in mind that there are three sorts of quarks. In general Yukawa Interactions are (remind them to construct the Lagragian and about the index sum)

$$L_{sH} = Y^u_{mn}\bar{Q}_m\bar{\varphi}U_n + Y^d_{mn}\bar{Q}_m\varphi D_n + h.c.$$ n=1,2,3 generations

$$U_n = u_R, c_R, t_R$$

$$D_n = d_R, s_R, b_R$$

$$\bar{\varphi}_\alpha = \varepsilon_{\alpha\beta}\varphi^{*\beta} \longrightarrow$$ Because singlets transform differently - $e^{iY\theta}$ - under U(1)

$$Q_n = \begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L$$

Unitary and mass gauge – mixing of quarks

Unitary gauge: $$\varphi = \begin{pmatrix} 0 \\ \frac{v+h(x)}{\sqrt{2}} \end{pmatrix}$$

Unitary gauge. Mass term NOT DIAGONAL in the generations

$$L_{mQ} = \frac{v}{\sqrt{2}} Y^u_{mn} \bar{u}_{L,m} u_{R,n} + \frac{v}{\sqrt{2}} Y^d_{mn} \bar{d}_{L,m} d_{R,n}$$

Interaction with W-boson – DIAGONAL in the generations

$$L_{WQ} = \frac{g}{\sqrt{2}} \bar{u}_{L,n} \hat{W}^+ d_{L,n} + h.c.$$

What is a particle?

There appears particle, for example u, but what is it mass?

Only a particle with a certain mass can propagate as a wave.

u, d – column of 3 components. Diagonalize mass matrix using

$$d_n = M_{d,nm} d'_m \quad \rightarrow d = M_d d'; \quad u = M_u u'$$

$\downarrow$

-matrixes $M_{u,d}$ we can find
From the condition

$$M_u^{-1} Y M_u$$
$$M_d^{-1} Y M_d$$

Mass
Matrixes
Are diagonal

$$L_{WQ} = \frac{g}{\sqrt{2}} \bar{u}_{L,n} \hat{W}^+ d_{L,n} + h.c.$$

$\updownarrow$

But then L_{WQ} not diagonal:

$$L_{WQ} = \frac{g}{\sqrt{2}} \bar{u}'_{L,m} \hat{W}^+ V_{nm} d'_{L,n} + h.c.$$

$$\boxed{V = M_u^{-1} M_d}$$

3x3 Cabibbo-Kobayashi-Maskawa mixing matrix

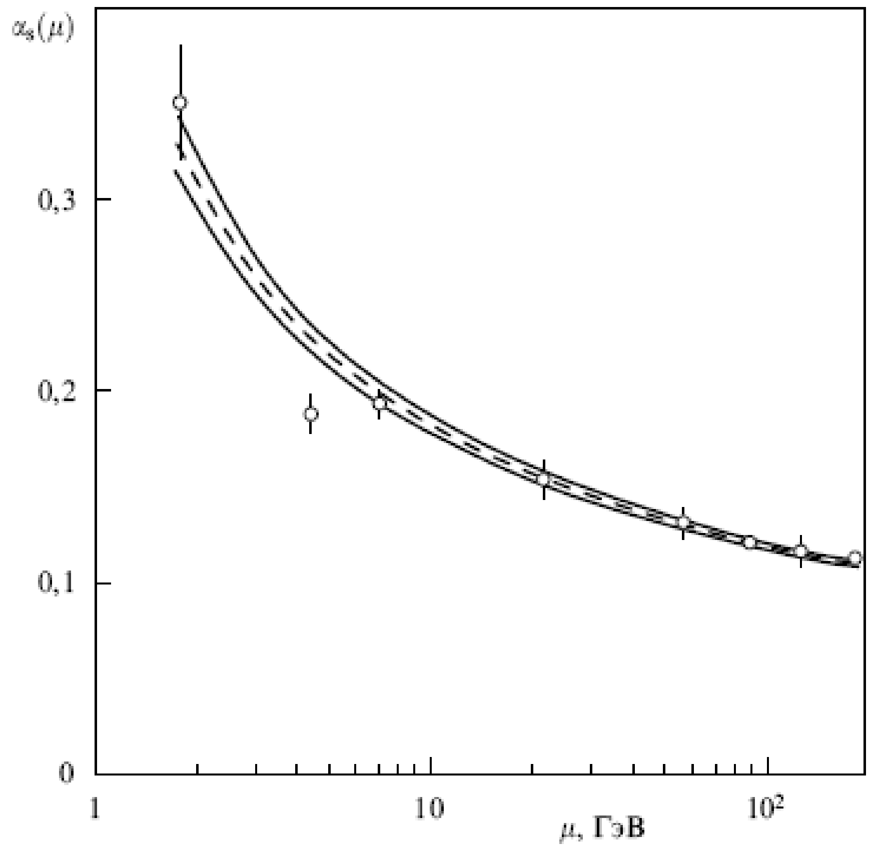


Рис. 1. Сила связи в КХД падает с ростом энергии.

Dependence of coupling constant of strong interaction on energy

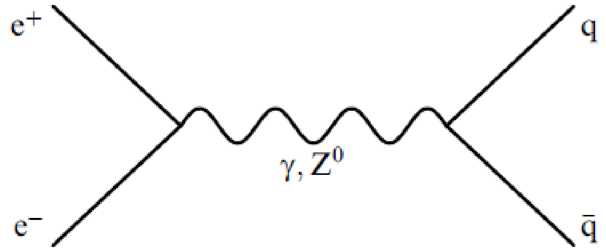


Рис. 2. Фейнмановская диаграмма для e^+e^--аннигиляции в кварк-антикварковую пару.

Electron-positron annihilation into quark-antiquark pair

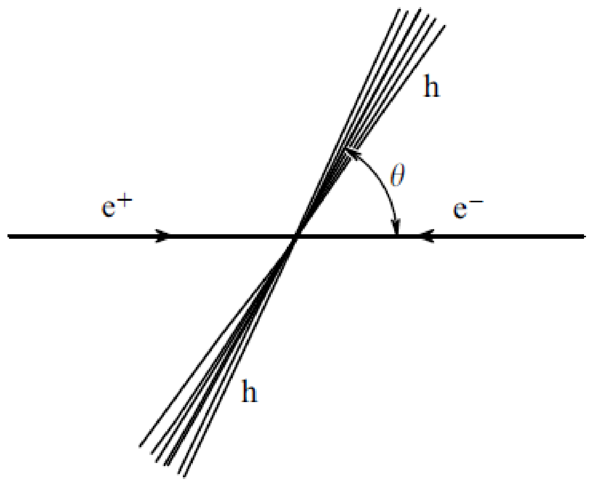
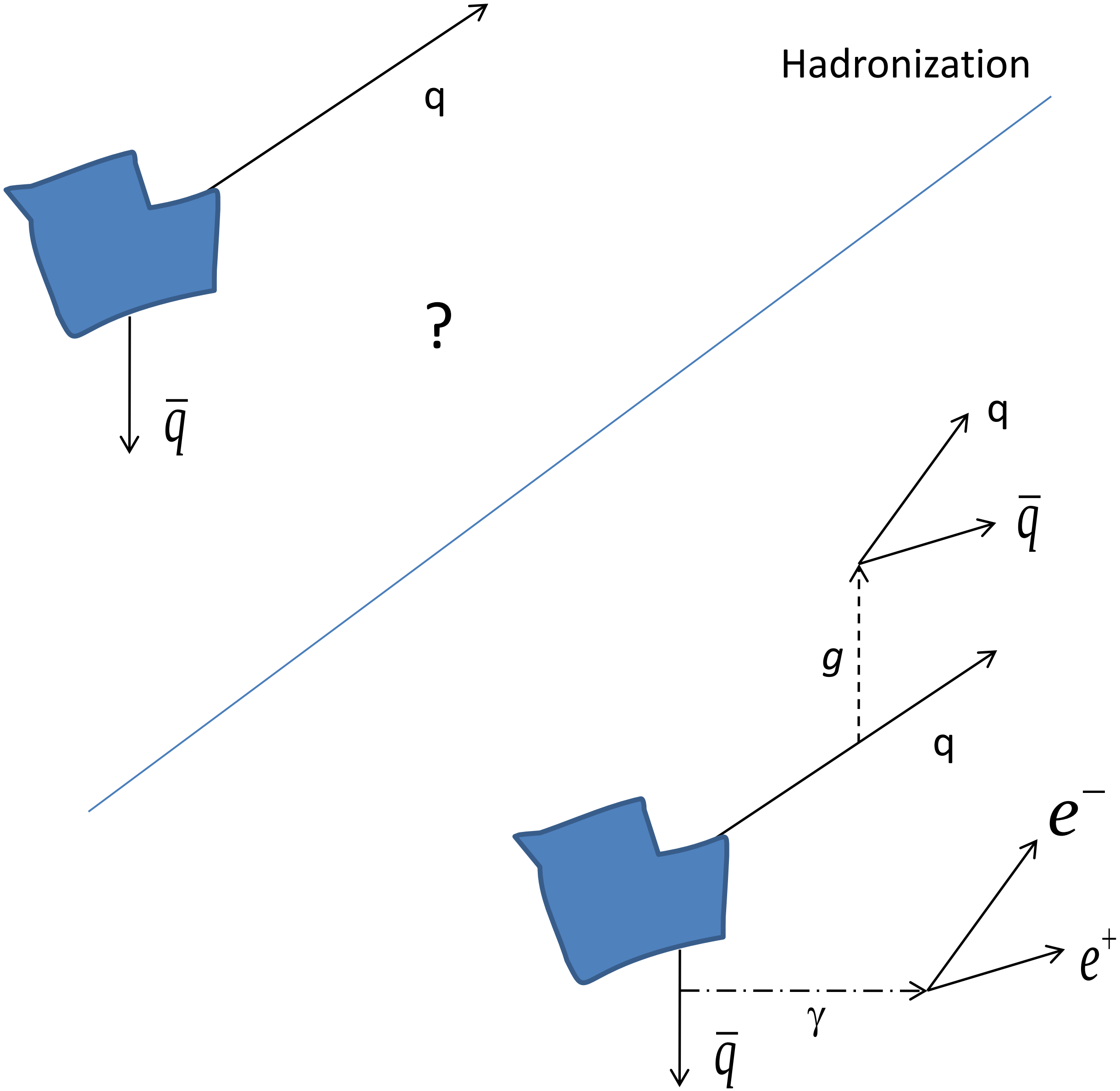


Рис. 3. Процесс превращения электрон-позитронной пары в две адронные струи.

Creation of two jets by electron-positron pair

$$R = \frac{\sigma_{\mathrm{tot}}^{e^+e^-\to h}}{\sigma^{e^+e^-\to\mu^+\mu^-}} = 3\sum_{q=1}^{n_\mathrm{f}} e_q^2$$



Hadronization
q
?
$\bar{q}$
q
$\bar{q}$
g
q
e^-
e^+
γ
$\bar{q}$

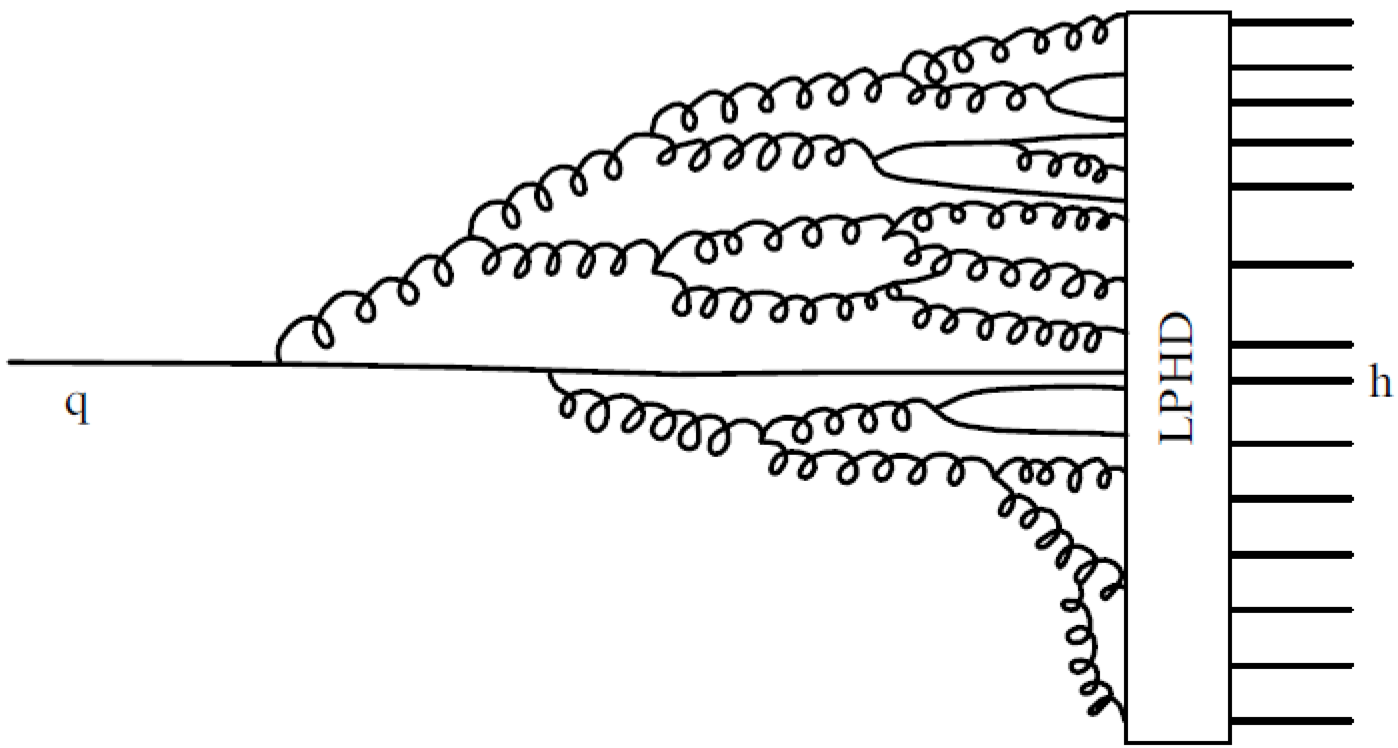


Рис. 4. Эволюция кварковой струи с учетом партонного этапа и адронизации.

Evolution of quark jet taking into account parton stage and hadronization

Confinement is the property that no isolated coloured charge can exist but only colour singlet particles. For example, the potential between a quark and an antiquark has been studied on the lattice and it has a Coulomb part at short distances and a linearly rising term at long distances:

$$V_{q\bar{q}} \approx C_F[\frac{\alpha_s(r)}{r} + ... + \sigma r] \tag{10}$$

where

$$\mathbf{1}_3\, C_F = \sum_A t^A t^A = \frac{N_C^2 - 1}{2N_C}\, \mathbf{1}_3 \tag{11}$$

Oscillation of the neutrino

Neutrinos are massive so the should be mixing of flavors as in the case of quarks

Oscillations happens when neutrino $|\nu_\alpha\rangle; \quad \alpha = e, \mu, \tau$

appears in process $W^+ \to e^+ \nu_e$

state of a certain mass propagates in space $|\nu_i\rangle; \quad i = 1, 2, 3$

$$L = \ldots M_{\alpha\beta} \bar{\nu}_\alpha \nu_\beta \ldots \xrightarrow{?} m_i \bar{\nu}_i \nu_i$$

Transition from gauge basis to mass one $|\nu_i\rangle = U_{\alpha i} |\nu_\alpha\rangle$

Obviously

$$M_{\alpha\beta} = \left(U M^{(m)} U^+ \right)_{\alpha\beta}; \quad M^{(m)}{}_{ij} = m_i \delta_{ij}$$

-Equation to find U и m_i

Since $$\nu_e = \sum_i U^+_{ei} \nu_i \quad \Rightarrow \quad W^+ \to \sum_i U^+_{ei} \nu_i e^+$$

Let the neutrino of sort α appear at the moment t. What is the probability to detect the neutrino of sort β at the distance L?

$$A_{\alpha\beta} = \int d^3x \psi^*_\beta(t,x+L)\psi_\alpha(0,x) = \int d^3x U^*{}_{\beta j}\psi^*_j(t,x+L)U_{\alpha k}\psi_k(0,x)$$

$$\psi^*_j(t,x+L) = e^{-iEt+ip_jL}\psi^*_j(0,x)$$

Solution of Dirac equation

$$A_{\alpha\beta} = e^{-iEt+ip_jL}U^*{}_{\beta j}U_{\alpha k}\int d^3x \psi^*_j(0,x)\psi_k(0,x) = e^{-iEt+ip_jL}U^*{}_{\beta j}U_{\alpha j}$$

$$= \sum_{j=1}^{3} e^{-i\frac{m_j^2}{2E}L} U_{\beta j}U_{\alpha j}$$

$$P = \left|A_{\alpha\beta}\right|^2$$

$$p_j = \sqrt{E^2 - m_j^2} \cong E - \frac{m_j^2}{2E}$$

Oscillations between two sorts of neutrino

$$U_{\alpha j} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

θ=?

Then the probability to change the flavor is

$$P_{\alpha\to\beta} = \left|A_{\alpha\to\beta}\right|^2 = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2}{4E}x\right)$$

Correct x $\xrightarrow{L}$

$$\frac{\Delta m^2}{4E}L_{osc} = \pi \to L_{osc} = \frac{4\pi E}{\Delta m^2}$$

We know in nature of the existence of six different types–**flavors**–of quarks: u, d, s, c, b, and t. Each flavor of quark is actually a triplet of fields, since the quarks transform irreducibly under the $SU(3)$ symmetry group that characterizes QCD. This $SU(3)$ symmetry is a local symmetry, so besides quarks in QCD one must introduce the $3^2 - 1 = 8$ gauge fields which are associated with the local $SU(3)$ symmetry. These 8 gauge fields are known as **gluons**, since they help bind quarks into hadrons— like protons and π-mesons.

Let $q^f_\alpha(x)$ stand for a quark field, with the index f denoting the various flavors $f = \{u, d, s, c, b, t\}$ and $\alpha = \{1, 2, 3\}$ being an $SU(3)$ index. Under local infinitesimal $SU(3)$ transformation then one has:

$$q^f_\alpha(x) \to q'^f_\alpha(x) = \left[\delta_{\alpha\beta} + i\delta a_i(x)\left(\frac{\lambda_i}{2}\right)_{\alpha\beta}\right] q^f_\beta(x) \ . \tag{138}$$

In the above, the λ_i matrices $i = 1, \ldots, 8$ are the 3×3 Gell-Mann matrices [9] transforming as the **3** representation of $SU(3)$. The $SU(3)$ structure constants–denoted here by f_{ijk}–are easily found by using the explicit form of the λ-matrices given below:

$$\lambda_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; \quad \lambda_2 = \begin{bmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; \quad \lambda_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix};$$

$$\lambda_4 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}; \quad \lambda_5 = \begin{bmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{bmatrix}; \quad \lambda_6 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}; \tag{139}$$

$$\lambda_7 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{bmatrix} \; ; \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

$$\left[\frac{\lambda_i}{2}\,, \quad \frac{\lambda_j}{2}\right] = i\; f_{ijk}\frac{\lambda_k}{2}$$

$$F_i^{\mu\nu} = \partial^\mu A_i^\nu - \partial^\nu A_i^\mu + g f_{ijk} A_j^\mu A_k^\nu$$

$$D^\mu_{\alpha\beta} q^f_\beta = \left[\partial^\mu \delta_{\alpha\beta} - ig\left(\frac{\lambda_i}{2}\right)_{\alpha\beta} A_i^\mu\right] q^f_\beta$$

Using the above equations, it is easy to see that the QCD Lagrangian

$$\mathcal{L}_{\mathrm{QCD}} = \sum_f -\bar{q}^f_\alpha \left[\gamma^\mu \frac{1}{i}(D_\mu)_{\alpha\beta} + m_f \delta_{\alpha\beta}\right] q^f_\beta - \frac{1}{4} F_i^{\mu\nu} F_{i\mu\nu} \tag{144}$$

is locally $SU(3)$ invariant. In the above, the parameters m_f are mass terms for each flavor f of quarks. If these terms were absent, that is if one could set $m_f \to 0$, it is clear that the QCD Lagrangian has a large global symmetry in which quarks of one flavor are changed into quarks of another flavor. For six flavors of quarks, it is not difficult to show that, in